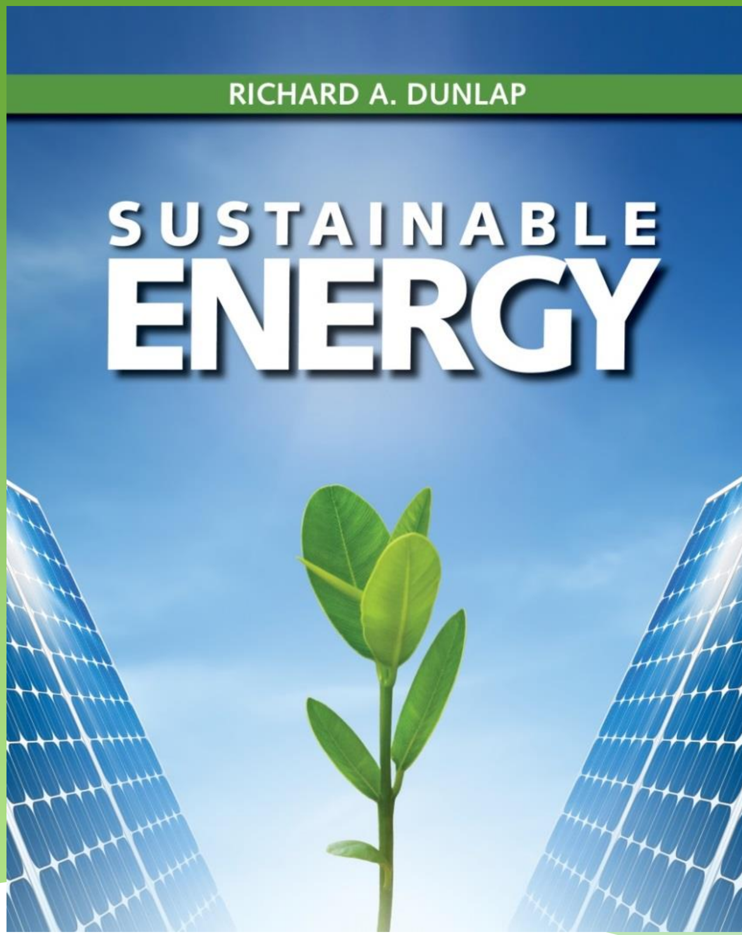
Sustainable Energy



Chapter 5

* Some Basic Nuclear Physics

Learning Objectives

- The composition and stability of the nucleus.
- The relationship of the binding energy of the nucleus to its properties.
- The statistics of nuclear decay processes.
- Alpha, beta, and gamma decay processes.
- The reactions between neutrons and nuclei.

Definition of a nuclide

A nuclear species (or nuclide) is a nucleus with a specific number of neutron, N, and a specific number of protons, Z.

A neutral atom of the nuclide would also contain Z atomic electrons.

Nucleons

Neutron and protons are collectively referred to as nucleons and the total number of nucleons,

$$A = N + Z \qquad (5.1)$$

is referred to as the mass number.

Isotopes

A nuclide is identified by the terminology ^{A}E where E is the name of the element.

An element can have several isotopes all of which have the same value of Z but different values of N.

The table shows examples of the isotopes of carbon (all with $Z = 6$)

Table 5.1: Summary of the properties of known carbon nuclei, $Z = 6$.

N	A	mass (u)	natural abundance (%)	half-life
3	9	9.031040087	0	127 ms
4	10	10.01685311	0	19.3 s
5	11	11.01143382	0	20.4 m
6	12	12.00000000	98.9	∞
7	13	13.00335484	1.1	∞
8	14	14.00324199	~0	5730 y
9	15	15.01059926	0	2.45 s
10	16	16.01470124	0	747 ms

Atomic mass unit

Masses are given as masses of neutral atoms in atomic mass units (u) where

$$1\ u = 1.6605402\times10^{-27}\ kg$$

Stable and unstable carbon isotopes

Carbon nuclei with 6 or 7 neutrons are stable.

Carbon nuclei with 3 to 6 or 7 to 10 neutrons are unstable.

Carbon nuclei with less than 3 or more than 10 neutrons are not observed.

The known stable and unstable nuclides as a function of *Z* are shown on a Segrè plot

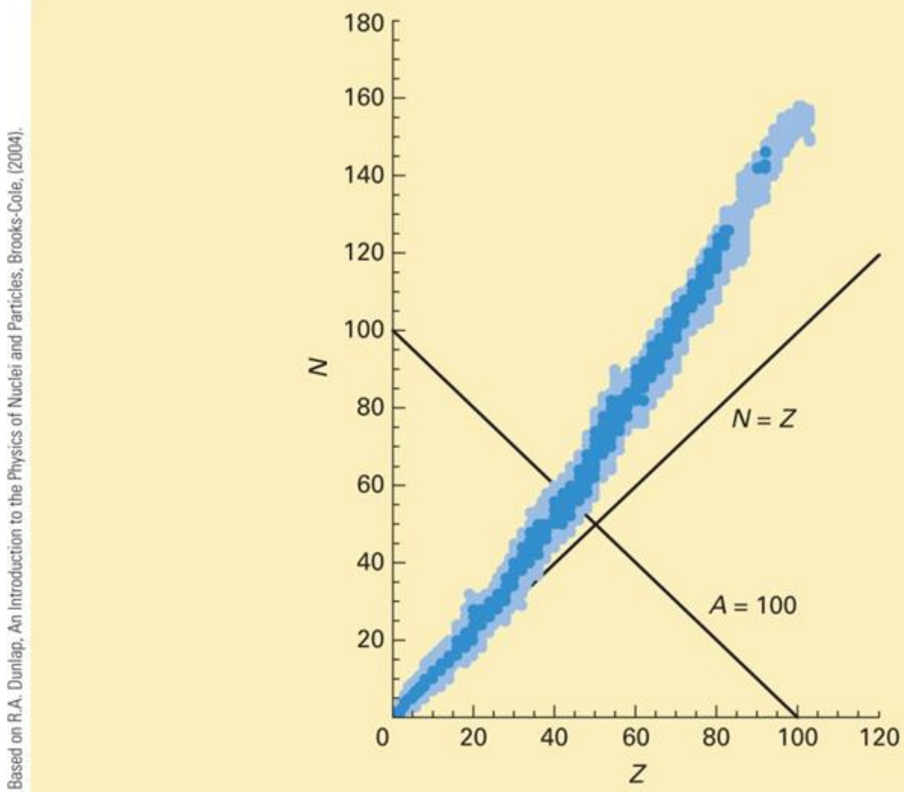


Figure 5.1: Segrè plot of stable nuclei (dark blue area) and unstable nuclei (light blue area).

Nuclear stability

Light nuclei are stable with $N \approx Z$
but heavy nuclei require $N > Z$ to be stable.

This is a crucial feature for the operation of a nuclear reactor.

Binding Energy

The neutrons and protons in the nucleus are held together by the nuclear binding energy.

Interactions in nature

The nuclear binding energy, *B*, is dominated by the strong interaction, which is one of the four fundamental interactions in nature

Table 5.2: The four known interactions in nature and some of their properties.

interaction	relative strength	range (m)	acts on
strong	1	10^{-15}	hadrons
electromagnetic	10^{-2}	long	charges
weak	10^{-5}	10^{-18}	leptons and hadrons
gravitational	10^{-39}	long	masses

Einstein's relation

Einstein's relation relates energy and mass

$$E = mc^2 \quad (5.2)$$

Mass equivalent of the binding energy

Binding energy is manifested as a component of the nuclear mass

$$m = \frac{B}{c^2} \qquad (5.3)$$

where *B* is defined to be explicitly positive.

Nuclear and atomic masses

The mass of a bound nucleus with *N* neutrons and Z protons will be

$$m_{\mathrm{nucleus}} = Nm_{\mathrm{n}} + Zm_{\mathrm{p}} - \frac{B}{c^2} \qquad \textbf{(5.4)}$$

The mass of a neutral atom also includes the electron mass and the electronic binding energy, *b*,

$$m_{\mathrm{atom}} = Nm_{\mathrm{n}} + Z(m_{\mathrm{p}} + m_{\mathrm{e}}) - \frac{B}{c^2} - \frac{b}{c^2} \qquad \textbf{(5.5)}$$

The masses and stabilities of nuclei with A = 49

Increasing binding energy leads to increased stability and decreased total mass.

Table 5.4: Atomic masses of nuclides with $A = 49$.

nuclide	N	Z	mass (u)	half-life	decay
^{49}Ca	29	20	48.9556733	8.8 m	β^-
^{49}Sc	28	21	48.95002407	57.5 m	β^-
^{49}Ti	27	22	48.94787079	∞	stable
^{49}V	26	23	48.94851691	330 d	ec
^{49}Cr	25	24	48.95134114	41.9 m	β^+, ec

Units in Einstein's relation

In Einstein's relation mass is commonly expressed in atomic mass units (u) and energy is commonly expressed in millions of electron volts (MeV).

So the speed of light squared is expressed in MeV/u with a value of

$$c^2 = 931.494 \text{ MeV/u}$$

Nuclear decays

Unstable nuclei decay according to the nuclear decay law

$$N(t) = N_0 e^{-\lambda t} \quad (5.14)$$

where $N(t)$ is the number of nuclei of a given species that exist at time t and N_0 is the number at time $t=0$.

Radioactive decay law

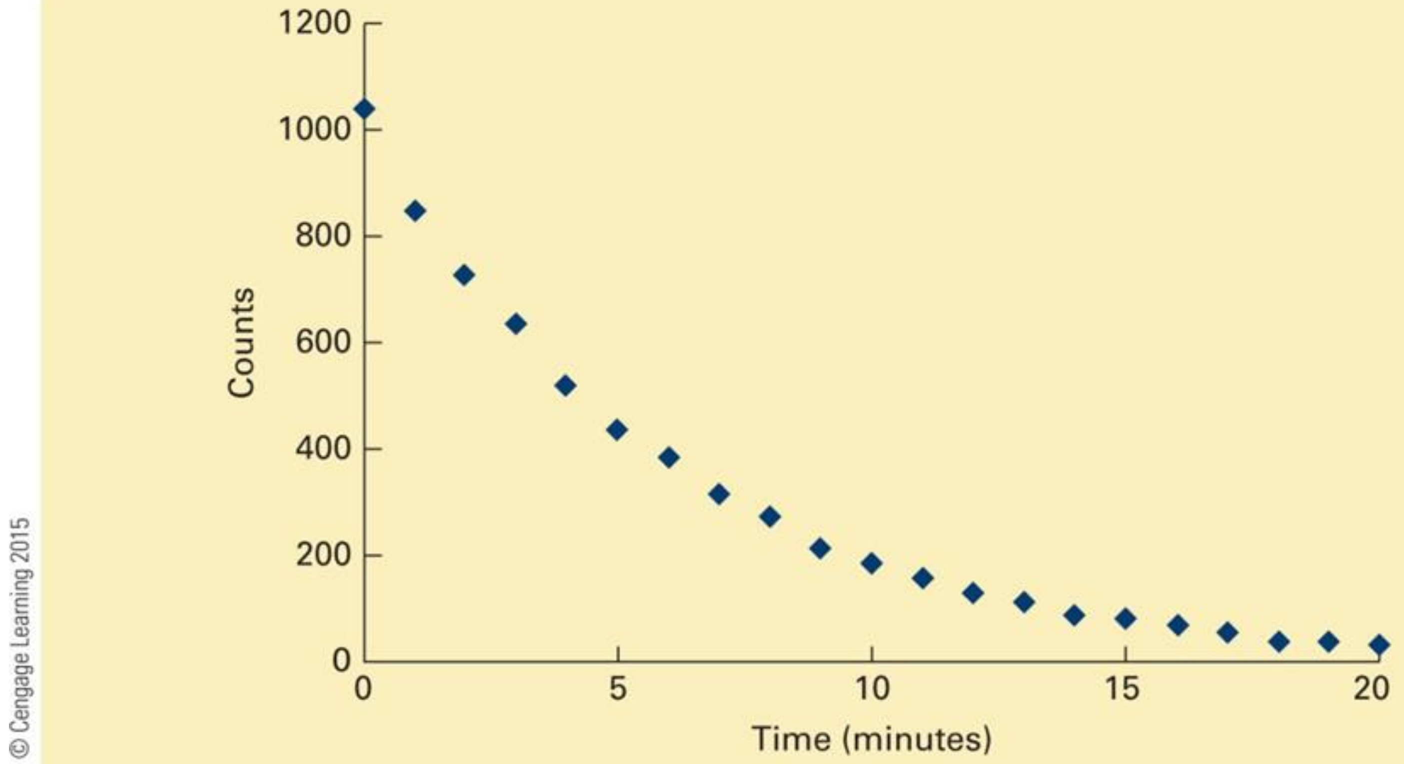


Figure 5.2: Measured decays per unit time for a radioactive source.

Nuclear decay processes (beta decay)

beta decay results from the weak interaction and changes a neutron to a proton (β^- decay) or vice versa (β^+ decay).

β⁻ decay

An example of β^- decay is

$$^{49}\mathrm{Sc} \rightarrow {}^{49}\mathrm{Ti} + \mathrm{e}^- + \overline{\nu}_\mathrm{e} \qquad (5.19)$$

where an electron and electron antineutrino are produced.

The nucleus on the left hand side is the parent nucleus and the nucleus on the right hand side is the progeny.

β^+ decay

An example of β^+ decay is

$$^{49}V \rightarrow {}^{49}Ti + e^+ + \nu_e \qquad (5.21)$$

where a positron (antielectron) and an electron neutrino are produced.

Energy from decay processes

Energy released in the decay (in the form of kinetic energy of the by-products) is given by Einstein's relation

$$E = \Delta mc^2 \tag{5.22}$$

An example is the β^- decay of ^{49}Sc

$$E = (48.95002407\ \mathrm{u} - 48.94787079\ \mathrm{u}) \times 931.494\ \mathrm{MeV/u} = 2.01\ \mathrm{MeV} \tag{5.23}$$

Alpha decay

Alpha decay results from to two neutrons and two protons escaping from a nucleus; e.g.

$$^{241}\text{Am} \rightarrow {}^{237}\text{Np} + \alpha \qquad \textbf{(5.24)}$$

where the alpha particle (α) is a bound system corresponding to a ^{4}He nucleus.

Segrè plots of decay processes

Different decay processes can be represented on a detailed portion of a Segrè plot.

The arrows show the decays

$$^{241}\mathrm{Am} \rightarrow {}^{237}\mathrm{Np} + \alpha \qquad (5.24)$$

$$^{235}\mathrm{Pa} \rightarrow {}^{235}\mathrm{U} + \mathrm{e}^{-} + \overline{\nu}_{\mathrm{e}} \qquad (5.25)$$

$$^{235}\mathrm{Np} \rightarrow {}^{235}\mathrm{U} + \mathrm{e}^{+} + \nu_{\mathrm{e}} \qquad (5.26)$$

α, β⁻ and β⁺ decays

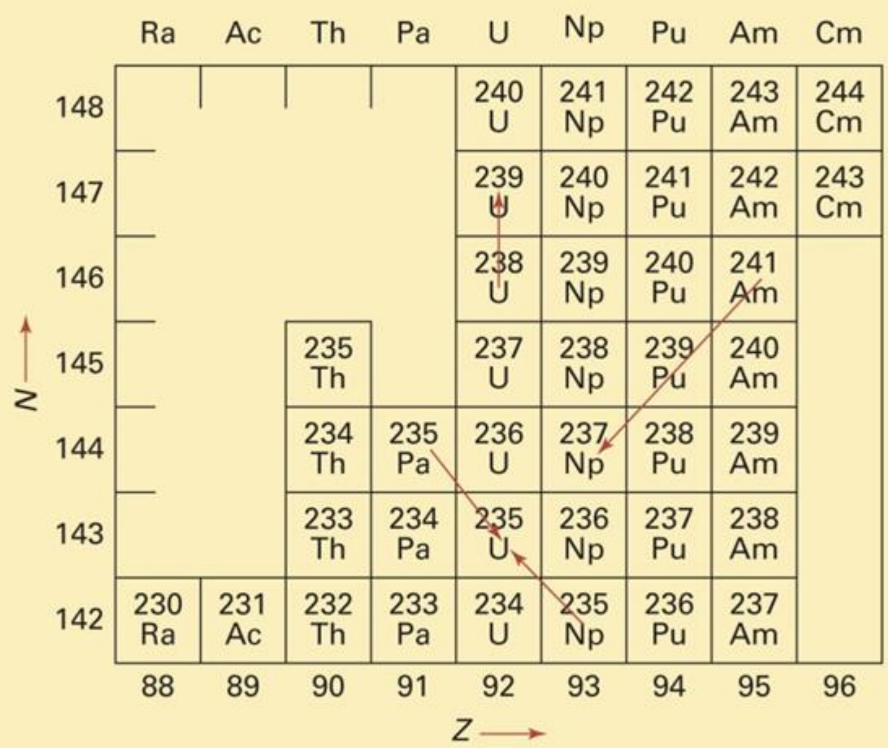


Figure 5.3: Region of the Segrè plot between N = 142 to 148 and Z = 88 to 96, corresponding to A = 230 to 244. An example of α-decay as given in equation (5.24) and examples of β^+ and β^- decays as given in equations (5.25) and (5.26), respectively are shown. Neutron capture by ^{238}U (discussed in the next section) is also illustrated.

Gamma decay

A nucleus in an excited state can decay to a lower energy level by emitting a photon (gamma ray or γ-ray).

In beta and alpha decay processes the progeny nucleus is often left in an excited state and subsequently decays by gamma decay.

Nuclear reactions

Nuclear reactions occur when a nucleus (A) is struck by a particle (a) resulting in the production of a nucleus (B) and a particle (b)

$$a + A \rightarrow B + b \qquad (5.27)$$

This is written as A(a,b)B.

The (n,p) reaction

An example of a nuclear reaction is

$$n + {}^{14}N \rightarrow {}^{14}C + p \qquad (5.29)$$

where the ^{14}N nucleus captured the incident neutron and emitted one of its protons leaving a ^{14}C nucleus behind.

Exothermic and endothermic reactions

Nuclear reactions (like chemical reactions) can be

exothermic (if they give up energy) or

endothermic (if they require energy to proceed)

The (n,γ) reaction

The (n,γ) reaction results when a nucleus captures a neutron and is left in an excited state which then decays by gamma emission. e.g.

$$n + {}^{238}U \rightarrow {}^{239}U + \gamma \quad (5.31)$$

This is an important reaction for nuclear reactors.

Summary

- The nucleus consists of neutrons and protons held together by the strong interaction
- Light nuclei have approximately equal numbers of neutrons and protons
- Heavy nuclei have more neutrons than protons
- Unstable nuclei can decay by beta decay, alpha decay or gamma decay
- Nuclear binding energy can be released during nuclear decays and nuclear reactions